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DISAMBIGUATION OF LAPLACE WEIERSTRASS TRANSFORM

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ABSTRACT

In this paper we discuss disambiguation of Laplace-Weierstrass transform which measures the sensitivity to change of one quantity to another quantity and the various result by using the same. The derivative of a function at a chosen input value describes the best linear approximation of the function near that input value. In this paper a technique involving integral and differential operators.

KEYWORDS: Laplace-Weierstrass Transform, Functions, Existence Theorem, Differential Operator

1. INTRODUCTION

Laplace-Weierstrass transform (LWT) is used in signal analysis, this suggest that if the signal f contains the frequency b, then the transformed signal F will contain the same frequency. The LW transform is defined as an integral transform of the initial temperature function, with the kernel as the source solution of the heat equation.

The Laplace-Weierstrass transform is used for solving differential and integral equations. In physics and engineering it is used for analysis of linear time-invariant systems such as electrical circuits, harmonic oscillators, optical devices and mechanical systems. This transform is related to other transforms such as Stieltjes transform, Fourier transform, Mellin transform, Z-transform, Borel transform etc.

In this paper we have to introduced the disambiguation of Laplace-Weierstrass transform i.e. existence of $f^{'}(t,y)$ and $f^{*}(t,y)$ and their values and also solve various results by using this. Derivatives are frequently used to find the maxima and minima.

The paper is organized as follows. Section [2] gives the testing function space of Laplace-Weierstrass transform. In section [3] theorem and some results are discussed. Lastly the conclusion is stated.

2. THE TESTING FUNCTION SPACE $LW_{a,b}$

$LW_{a,b}$ as the linear space of all complex valued smooth functions $\phi(t,y)$ on $0<t<\infty$, $0<y<\infty$ such that for each $p, q = 0, 1, 2, ---$

$$\gamma_{a,b,p,q}\,\phi(t,y) = \sup_{\substack{0<t<\infty \\ 0<y<\infty}} \left| e^{at-\frac{by}{2}+\frac{y^2}{4}} D^{p+q}\phi(t,y) \right| < \infty,$$

for some fixed number a, b in R (2.1)

The space $LW_{a,b}$ is complete and a Frechet space. This topology is generated by the total families of countably multinorms space given by (2.1).

3. DISAMBIGUATION THEOREM

Let $f(t,y)$ be continuous for all $(t,y)\geq 0$ and be of exponential order as $t,y\to\infty$ and $f^{'}(t,y)$ *and* $f^{*}(t,y)$ *are* of class A, then Laplace-Weierstrass transform of the derivative $f^{'}(t,y)$ *and* $f^{*}(t,y)$ and their values exists.

Proof: Here given that the functions $f^{'}(t,y)$ *and* $f^{*}(t,y)$ *are* piecewise continuous on every finite interval in the range $(t,y)\geq 0$.

$\Rightarrow e^{-st-\frac{(x-y)^2}{4}} f^{'}(t,y)$ *and* $e^{-st-\frac{(x-y)^2}{4}} f^{*}(t,y)$ *are* R-integrable over any finite interval in the range $(t,y)\geq 0$.

Now definition of LW transform for $f^{'}(t,y)$ is given by,

$$LW\left\{ f^{'}(t,y)\right\}=\frac{1}{\sqrt{4\pi}}\int_0^\infty\int_0^\infty e^{-st-\frac{(x-y)^2}{4}} f^{'}(t,y)\,dt\,dy$$

where f ' is the differentiation of f w. r. to ' t '.

$$=-\frac{1}{\sqrt{4\pi}}\int_0^\infty e^{\frac{-(x-y)^2}{4}} f(0,y)\,dy+s\,LW\left\{ f(t,y)\right\} \tag{3.1}$$

Case I: When $f(0,y)=e^{by}$ then equation (3.1) becomes

$$=\frac{-1}{\sqrt{4\pi}}\int_0^\infty e^{\frac{-(x-y)^2}{4}} e^{by}dy+s\,LW\left\{e^{at+by}\right\}$$

$$=e^{bx+4b^2}erf_c\left(\frac{x}{4}+2b\right)-\frac{s\,e^{bx+4b^2}}{s-a}erf_c\left(\frac{x}{4}+2b\right)$$

$$=-\frac{a\,e^{bx+4b^2}}{s-a}erf_c\left(\frac{x}{4}+2b\right) \tag{3.2}$$

Case II: When $f(0,y)=1$ then equation (3.1) becomes

$$=\frac{-1}{\sqrt{4\pi}}\int_0^\infty e^{\frac{-(x-y)^2}{4}} dy+s\,LW\left\{1\right\}$$

$$= erf_c\left(\frac{x}{4}\right) - erf_c\left(\frac{x}{4}\right)$$

$$= 0 \tag{3.3}$$

Case III: When $f(o, y) = \sin by$ then equation (3.1) becomes

$$= \frac{-1}{\sqrt{4\pi}} \int_0^\infty e^{\frac{-(x-y)^2}{4}} \sin by \; dy + s\, LW\{\sin(at + by)\}$$

$$= \frac{1}{2i}\left\{ e^{ibx-4b^2} erf_c\left(\frac{x}{4} + 2ib\right) - e^{-ibx-4b^2} erf_c\left(\frac{x}{4} - 2ib\right)\right\}$$

$$+ \frac{si}{2}\left[\frac{e^{ibx-4b^2}}{s - ia} erf_c\left(\frac{x}{4} + 2ib\right) - \frac{e^{-ibx-4b^2}}{s + ia} erf_c\left(\frac{x}{4} - 2ib\right)\right]$$

$$= \frac{-a}{2(s-ia)} e^{ibx-4b^2} erf_c\left(\frac{x}{4} + 2ib\right) - \frac{a}{2(s+ia)} e^{-ibx-4b^2} erf_c\left(\frac{x}{4} - 2ib\right) \tag{3.4}$$

Case IV: When $f(0, y) = \cos by$,then equation (3.1) becomes

$$= \frac{-1}{\sqrt{4\pi}} \int_0^\infty e^{\frac{-(x-y)^2}{4}} \cos by \; dy + s\, LW\{\cos(at + by)\}$$

$$= \frac{1}{2}\left\{ e^{ibx-4b^2} erf_c\left(\frac{x}{4} + 2ib\right) + e^{-ibx-4b^2} erf_c\left(\frac{x}{4} - 2ib\right)\right\}$$

$$- \frac{s}{2}\left[\frac{e^{ibx-4b^2}}{s - ia} erf_c\left(\frac{x}{4} + 2ib\right) + \frac{e^{-ibx-4b^2}}{s + ia} erf_c\left(\frac{x}{4} - 2ib\right)\right]$$

$$= \frac{-ia}{2(s-ia)} e^{ibx-4b^2} erf_c\left(\frac{x}{4} + 2ib\right) + \frac{ai}{2(s+ia)} e^{-ibx-4b^2} erf_c\left(\frac{x}{4} - 2ib\right) \tag{3.5}$$

Case V: When $f(0, y) = \sinh by$, then equation (3.1) becomes

$$= \frac{-1}{\sqrt{4\pi}} \int_0^\infty e^{\frac{-(x-y)^2}{4}} \sinh by \; dy + s\, LW\{\sinh(at + by)\}$$

$$=\frac{1}{2}\left\{e^{bx+4b^2}erf_c\left(\frac{x}{4}+2b\right)-e^{-bx+4b^2}erf_c\left(\frac{x}{4}-2b\right)\right\}$$

$$-\frac{s}{2}\left[\frac{e^{bx+4b^2}}{s-a}erf_c\left(\frac{x}{4}+2b\right)-\frac{e^{-bx+4b^2}}{s+a}erf_c\left(\frac{x}{4}-2b\right)\right]$$

$$=\frac{-a}{2(s-a)}e^{bx+4b^2}erf_c\left(\frac{x}{4}+2b\right)-\frac{a}{2(s+a)}e^{-bx+4b^2}erf_c\left(\frac{x}{4}-2b\right) \tag{3.6}$$

Case VI: When $f(0,y)=\cosh by$, then equation (3.1) becomes

$$=\frac{-1}{\sqrt{4\pi}}\int_0^\infty e^{\frac{-(x-y)^2}{4}}\cosh by\,dy+s\,LW\{\cosh(at+by)\}$$

$$=\frac{1}{2}\left\{e^{bx+4b^2}erf_c\left(\frac{x}{4}+2b\right)+e^{-bx+4b^2}erf_c\left(\frac{x}{4}-2b\right)\right\}$$

$$-\frac{s}{2}\left[\frac{e^{bx+4b^2}}{s-a}erf_c\left(\frac{x}{4}+2b\right)+\frac{e^{-bx+4b^2}}{s+a}erf_c\left(\frac{x}{4}-2b\right)\right]$$

$$=\frac{-a}{2(s-a)}e^{bx+4b^2}erf_c\left(\frac{x}{4}+2b\right)+\frac{a}{2(s+a)}e^{-bx+4b^2}erf_c\left(\frac{x}{4}-2b\right) \tag{3.7}$$

And now definition of LW transform for $f^*(t,y)$ is given by,

$$LW\{f^*(t,y)\}=\frac{1}{\sqrt{4\pi}}\int_0^\infty\int_0^\infty e^{-st-\frac{(x-y)^2}{4}}f^*(t,y)\,dy\,dt$$

where f is the differentiation of f w. r. to 'y'

$$=-\frac{1}{\sqrt{4\pi}}\int_0^\infty e^{-st-\frac{x^2}{4}}f(t,0)\,dt-\frac{x}{2}LW\{f(t,y)\}+\frac{1}{2}LW\{y\,f(t,y)\} \qquad (3.1^*)$$

Case I: When $f(t,0)=e^{at}$ then equation (3.1*) becomes

$$=\frac{-1}{\sqrt{4\pi}}\int_0^\infty e^{-st-\frac{x^2}{4}}e^{at}dt-\frac{x}{2}LW\{e^{at+by}\}+\frac{1}{2}LW\{y\,e^{at+by}\}$$

$$= \frac{-e^{\frac{-x^2}{4}}}{(s-a)\sqrt{4\pi}} + \frac{1}{\sqrt{4\pi}(s-a)}\left[e^{\frac{-x^2}{4}} + \sqrt{\pi}(x+2b)e^{bx+4b^2} erf_c\left(\frac{x}{4}+2b\right)\right]$$

$$+ \frac{x\, e^{bx+4b^2} erf_c\left(\frac{x}{4}+2b\right)}{2(s-a)}$$

$$= (x+2b)\frac{e^{bx+4b^2}}{s-a} erf_c\left(\frac{x}{4}+2b\right) \tag{3.2*}$$

Case II: When $f(t,0)=1$ then equation (3.1*) becomes

$$= \frac{-1}{\sqrt{4\pi}}\int_0^{\infty} e^{-st-\frac{x^2}{4}} dt - \frac{x}{2} LW\{1\} + \frac{1}{2} LW\{y.1\}$$

$$= -\frac{1}{s\sqrt{4\pi}} e^{-\frac{x^2}{4}} + \frac{x\sqrt{\pi}}{s\sqrt{4\pi}} erf_c\left(\frac{x}{4}\right) - \frac{1}{\sqrt{4\pi}}\left[\frac{-e^{\frac{-x^2}{4}}}{s} + \frac{x\sqrt{\pi}}{s} erf_c\left(\frac{x}{4}\right)\right]$$

$$= 0 \tag{3.3*}$$

Case III: When $f(t,0)=\sin at$ then equation (3.1*) becomes

$$= \frac{-1}{\sqrt{4\pi}}\int_0^{\infty} e^{\frac{-x^2}{4}-st} \sin at\ dt - \frac{x}{2} LW\{\sin(at+by)\} + \frac{1}{2} LW\{y\sin(at+by)\}$$

$$-\frac{xi}{4}\left[\frac{e^{ibx-4b^2} erf_c\left(\frac{x}{4}+2ib\right)}{s-ia}\right] - \frac{1}{2i\sqrt{4\pi}}\left[\frac{(x+2ib)\sqrt{\pi}e^{ibx-4b^2} erf_c\left(\frac{x}{4}+2ib\right)}{s-ia}\right]$$

$$+\frac{xi}{4}\left[\frac{e^{-ibx-4b^2} erf_c\left(\frac{x}{4}-2ib\right)}{s+ia}\right] + \frac{1}{2i\sqrt{4\pi}}\left[\frac{(x-2ib)\sqrt{\pi}e^{-ibx-4b^2} erf_c\left(\frac{x}{4}-2ib\right)}{s+ia}\right]$$

$$= \frac{-b}{2}\left[\frac{e^{ibx-4b^2} erf_c\left(\frac{x}{4}+2ib\right)}{s-ia} + \frac{e^{-ibx-4b^2} erf_c\left(\frac{x}{4}-2ib\right)}{s+ia}\right] \tag{3.4*}$$

Case IV: When $f(t,0)=\cos at$, then equation (3.1*) becomes

$$=\frac{-1}{\sqrt{4\pi}}\int_0^\infty e^{-st-\frac{x^2}{4}}\cos at\ dt-\frac{x}{2}LW\{\cos(at+by)\}+\frac{1}{2}LW\{y\cos(at+by)\}$$

$$=\frac{-se^{\frac{-x^2}{4}}}{\sqrt{4\pi}(s^2+a^2)}+\frac{x}{4}\left[\frac{e^{ibx-4b^2}erf_c\left(\frac{x}{4}+2ib\right)}{(s-ia)}+\frac{e^{-ibx-4b^2}erf_c\left(\frac{x}{4}-2ib\right)}{(s+ia)}\right]$$

$$+\frac{se^{\frac{-x^2}{4}}}{\sqrt{4\pi}(s^2+a^2)}-\frac{1}{2\sqrt{4\pi}}\left[\frac{(x+2ib)\sqrt{\pi}e^{ibx-4b^2}erf_c\left(\frac{x}{4}+2ib\right)}{(s-ia)}+\frac{(x-2ib)\sqrt{\pi}e^{-ibx-4b^2}erf_c\left(\frac{x}{4}-2ib\right)}{(s+ia)}\right]$$

$$=\frac{-ib}{2(s-ia)}e^{ibx-4b^2}erf_c\left(\frac{x}{4}+2ib\right)+\frac{bi}{2(s+ia)}e^{-ibx-4b^2}erf_c\left(\frac{x}{4}-2ib\right) \tag{3.5*}$$

Case V: When $f(t,0)=\sinh at$, then equation (3.1*) becomes

$$=\frac{-1}{\sqrt{4\pi}}\int_0^\infty e^{-st-\frac{x^2}{4}}\sinh at\ dt-\frac{x}{2}LW\{\sinh(at+by)\}+\frac{1}{2}LW\{y\sinh(at+by)\}$$

$$=\frac{-ae^{\frac{-x^2}{4}}}{\sqrt{4\pi}(s^2-a^2)}+\frac{x}{4}\left[\frac{e^{bx+4b^2}erf_c\left(\frac{x}{4}+2b\right)}{(s-a)}-\frac{e^{-bx+4b^2}erf_c\left(\frac{x}{4}-2b\right)}{(s+a)}\right]$$

$$+\frac{1}{2\sqrt{4\pi}}\left[\frac{e^{\frac{-x^2}{4}}+(x+2b)\sqrt{\pi}e^{bx+4b^2}erf_c\left(\frac{x}{4}+2b\right)}{(s-a)}+\frac{e^{\frac{-x^2}{4}}+(x-2b)\sqrt{\pi}e^{-bx+4b^2}erf_c\left(\frac{x}{4}-2b\right)}{(s+a)}\right]$$

$$=\frac{e^{\frac{-x^2}{4}}}{\sqrt{4\pi}(s+a)}+\left[\frac{x+b}{2(s-a)}\right]e^{bx+4b^2}erf_c\left(\frac{x}{4}+2b\right)-\frac{b}{2(s+a)}e^{-bx+4b^2}erf_c\left(\frac{x}{4}-2b\right) \tag{3.6*}$$

Case VI: When $f(t,0) = \cosh at$, then equation (3.1*) becomes

$$= \frac{-1}{\sqrt{4\pi}} \int_0^\infty e^{-st-\frac{x^2}{4}} \cosh at \, dt - \frac{x}{2} LW\{\cosh(at+by)\} + \frac{1}{2} LW\{ y\cosh(at+by) \}$$

$$= \frac{-se^{\frac{-x^2}{4}}}{\sqrt{4\pi}(s^2-a^2)} + \frac{x}{4}\left[\frac{e^{bx+4b^2} erf_c\left(\frac{x}{4}+2b\right)}{(s-a)} - \frac{e^{-bx+4b^2} erf_c\left(\frac{x}{4}-2b\right)}{(s+a)}\right]$$

$$+\frac{1}{2\sqrt{4\pi}}\left[\frac{e^{\frac{-x^2}{4}} + (x+2b)\sqrt{\pi}e^{bx+4b^2} erf_c\left(\frac{x}{4}+2b\right)}{(s-a)} - \frac{e^{\frac{-x^2}{4}} + (x-2b)\sqrt{\pi}e^{-bx+4b^2} erf_c\left(\frac{x}{4}-2b\right)}{(s+a)}\right]$$

$$= \frac{e^{\frac{-x^2}{4}}}{\sqrt{4\pi}(s+a)} + \left[\frac{x+b}{2(s-a)}\right] e^{bx+4b^2} erf_c\left(\frac{x}{4}+2b\right) - \frac{b}{2(s+a)} e^{-bx+4b^2} erf_c\left(\frac{x}{4}-2b\right) \tag{3.7*}$$

4. CONCLUSIONS

In this paper we have seen that how we obtained the derivative of Laplace-Weierstrass transform and also the results of various functions. In this paper a technique involving integral and differential operators has been used to effect the transform. Since this transform is an important tool in signal processing and many other branches of engineering, it provides new aspects to many mathematical disciplines such as transform theory, functional analysis, differential equation etc.

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